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饱和分数导数型粘弹性土-深埋圆形隧洞衬砌系统的动力特性

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摘 要: 在频率域研究了分数导数型粘弹性饱和土体和深埋圆形隧洞弹性衬砌相互作用的耦合简谐振动。将土骨架视为具有分数阶导数型本构的粘弹性体, 基于饱和多孔介质理论和平面弹性理论, 分别给出了饱和粘弹性土、弹性衬砌简谐振动的解析解。通过弹性衬砌和饱和土接触面处的连续性条件和衬砌内边界条件, 得到了饱和粘弹性土-衬砌系统的稳态动力响应, 给出了衬砌和饱和粘弹性土位移、应力和孔隙压力的解析表达式。在此基础上, 进行了参数研究, 讨论了物理和几何参数对系统响应的影响。研究表明: 饱和分数导数型粘弹性土-衬砌系统的动力响应与经典饱和弹性/粘弹性土-衬砌系统的动力响应差异很大。

关键词: 分数导数模型; 粘弹性饱和土; 圆形隧洞; 弹性衬砌; 参数研究

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DYNAMIC CHARACTERISTICS OF SATURATED FRACTIONAL DERIVATIVE TYPE VISCOELASTIC SOIL AND LINING SYSTEM WITH A DEEPLY EMBEDDED CIRCULAR TUNNEL

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Abstract: The coupled harmonic vibration of the saturated fractional derivative type viscoelastic soil and elastic lining of a deeply embedded circular tunnel is investigated in the frequency domain. Regarding the soil skeleton as a viscoelastic medium with a fractional derivative constitutive relation and based on the theory of saturated porous medium and plane elasticity, analytical solutions of the harmonic vibrations of the saturated viscoelastic soil and the elastic lining are presented, respectively. The steady-state dynamic responses of the saturated viscoelastic soil and lining system are obtained by the inner boundary conditions of the lining and the continuity conditions on the interface between the saturated soil and the lining, and the analytical expressions of the displacements, stresses and the pore water pressures for the saturated viscoelastic soil and lining are given. Then, the parameter study is conducted, and the influences of the physical and geometry parameter on the dynamic characteristics of the system are examined. It is revealed that there exist remarkable differences for the dynamic responses between the saturated fractional derivative type viscoelastic soil-lining system and the classical saturated elastic/viscoelastic soil-lining system.

Key words: fractional derivative model; saturated viscoelastic soil; circular tunnel; elastic lining; parameter study

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具有深埋隧洞的饱和粘弹性土体动力响应分析是岩土工程领域关注的课题之一,不仅具有重要的理论意义,而且在矿物勘探、地下隧洞等中得到了广泛应用。目前,对于此类问题的研究主要基于饱和土的 Biot 模型, Carter 和 Booker^[1-2]研究了边界透水和不透水两种情况下深埋圆形隧洞饱和土体的应力、位移和孔隙水压力分布。利用 Laplace 变换, Kumar 等^[3]给出了透水条件下深埋圆形隧洞饱和弹性土体轴对称动力分析的一般解法,考察了脉冲载荷作用下土体的动力响应。针对实际工程中隧洞边界的部分透水情况, Li^[4]和刘干斌等^[5]分析了部分透水条件下深埋隧洞饱和粘弹性土体的动力学行为,而 Lu 和 Jeng^[6]研究了轴对称载荷作用下饱和土体的动力响应,考察了沿隧洞轴向的土体力学行为。然而,上述研究忽略了隧洞衬砌对土体动力响应的影响。为此,将衬砌视为弹性壳, Zakout 和 Akkas^[7]研究了弹性土-衬砌系统的耦合动力响应,而刘干斌等^[8]分析了饱和粘弹性土-衬砌系统的动力特性。

由于经典微分型和积分型粘弹性本构关系需要较多的材料参数,试验确定较为困难, Bagley 和 Torvik^[9]首先建立了分数导数型粘弹性本构关系的理论基础,此后,分数导数型粘弹性本构关系得到了广泛深入的研究和应用^[10-13]。由于饱和多孔介质理论^[14]不仅严格满足连续介质力学的基本公理,避免了模型中的非相容性^[15],而且可包含诸如几何非线性和材料非线性等因素,目前已成功应用于各种实际问题的研究中^[16-18]。

基于饱和多孔介质理论和分数导数型粘弹性本构,刘林超和杨晓^[19]研究了饱和粘弹性土中桩的纵向振动问题。本文利用分数导数型粘弹性本构刻画土骨架的应力-应变关系,在频率域内研究了饱和粘弹性土体-深埋圆形隧洞弹性衬砌系统的动力特性,讨论了衬砌厚度、分数导数阶数、材料参数等对系统动力响应的影响,揭示了饱和分数导数型粘弹性土-衬砌系统与经典饱和弹性/粘弹性土-衬砌系统动力性态之间的差异。

1 粘弹性饱和土体的动力响应

如图 1 所示,设流相和固相体积分数分别为 n^F 和 n^S 的无限饱和粘弹性土体中有无限长圆形隧洞,隧洞周边有厚为 d 、内半径为 R 的弹性衬砌,其内边界承受圆频率为 ω 的径向均布简谐激振力 $q_0 e^{i\omega t}$

($i^2 = -1$)的作用。记土骨架和孔隙水的真实质量密度分别为 ρ^{SR} 和 ρ^{FR} , 则其表观密度分别为 $\rho^S = n^S \rho^{SR}$ 和 $\rho^F = n^F \rho^{FR}$ 。假设在饱和粘弹性土-衬砌系统的谐振中,土骨架和衬砌为小变形,且衬砌与土完全紧密接触,即衬砌与饱和土在接触面上满足位移和应力连续。

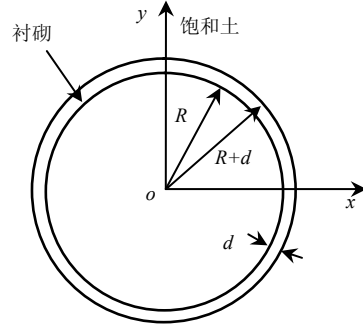


图 1 饱和粘弹性土体中的圆形隧洞

Fig.1 Circular tunnel in saturated viscoelastic soil

显然,该问题可简化为轴对称平面应变问题。设土骨架和孔隙水微观不可压,忽略其体积力和孔隙水的粘性,根据饱和多孔介质理论^[14],极坐标下饱和粘弹性土体的运动方程为:

$$\begin{cases} \frac{\partial T_{rr}^{SE}}{\partial r} + \frac{T_{rr}^{SE} - T_{\theta\theta}^{SE}}{r} - n^S \frac{\partial p}{\partial r} - \rho^S \frac{\partial^2 u_r^S}{\partial t^2} + S_v \left(\frac{\partial u_r^F}{\partial t} - \frac{\partial u_r^S}{\partial t} \right) = 0 \\ n^F \frac{\partial p}{\partial r} + \rho^F \frac{\partial^2 u_r^F}{\partial t^2} + S_v \left(\frac{\partial u_r^F}{\partial t} - \frac{\partial u_r^S}{\partial t} \right) = 0 \\ \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(n^S \frac{\partial u_r^S}{\partial t} + n^F \frac{\partial u_r^F}{\partial t} \right) \right] = 0 \end{cases} \quad (1)$$

其中: u_r^S 和 u_r^F 分别为土骨架和孔隙水的径向位移; 而 T_{rr}^{SE} 和 $T_{\theta\theta}^{SE}$ 为土骨架的有效应力; p 为孔隙水压力; $S_v = (n^F)^2 \gamma^{FR} / k^F$ 为表征流固两相的相互作用系数,而 γ^{FR} 为流相的容重, k^F 为 Darcy 渗流系数。

设粘弹性土骨架本构方程满足分数导数型微分关系^[20], 即:

$$\begin{cases} (1 + \tau_\varepsilon^\alpha D^\alpha) T_{rr}^{SE} = (1 + \tau_\sigma^\alpha D^\alpha) \times \left[\lambda^S \left(\frac{\partial u_r^S}{\partial r} + \frac{u_r^S}{r} \right) + 2\mu^S \frac{\partial u_r^S}{\partial r} \right] \\ (1 + \tau_\varepsilon^\alpha D^\alpha) T_{\theta\theta}^{SE} = (1 + \tau_\sigma^\alpha D^\alpha) \times \left[\lambda^S \left(\frac{\partial u_r^S}{\partial r} + \frac{u_r^S}{r} \right) + 2\mu^S \frac{u_r^S}{r} \right] \end{cases} \quad (2)$$

其中: λ^S 、 μ^S 、 τ_ε^α 和 τ_σ^α 为土骨架材料参数, $\lambda^S = 2\nu^S \mu^S / (1 - 2\nu^S)$, ν^S 为泊松比; $0 < \alpha < 1$; $D^\alpha = d^\alpha / dt^\alpha$ 为 α 阶 Riemann-Liouville 分数阶导数, 定义为:

$$D^\alpha[x(t)] = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{x(\tau)}{(t-\tau)^\alpha} d\tau \quad (3)$$

这里, $\Gamma(u) = \int_0^\infty t^{u-1} e^{-t} dt$ 为 Gamma 函数。

将式(2)代入土体运动方程式(1), 得控制方程:

$$\left\{ \begin{aligned} (1 + \tau_\sigma^\alpha D^\alpha) \left[(\lambda^S + 2\mu^S) \frac{\partial}{\partial r} \left(\frac{\partial u_r^S}{\partial r} + \frac{u_r^S}{r} \right) \right] - \\ (1 + \tau_\varepsilon^\alpha D^\alpha) \left[n^S \frac{\partial p}{\partial r} + \rho^S \frac{\partial^2 u_r^S}{\partial t^2} - \right. \\ \left. S_v \left(\frac{\partial u_r^F}{\partial t} - \frac{\partial u_r^S}{\partial t} \right) \right] = 0 \\ n^F \frac{\partial p}{\partial r} + \rho^F \frac{\partial^2 u_r^F}{\partial t^2} + S_v \left(\frac{\partial u_r^F}{\partial t} - \frac{\partial u_r^S}{\partial t} \right) = 0 \\ \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(n^S \frac{\partial u_r^S}{\partial t} + n^F \frac{\partial u_r^F}{\partial t} \right) \right] = 0 \end{aligned} \right. \quad (4)$$

对于饱和粘弹性土-衬砌系统的简谐振动, 可设:

$$u_r^S = R U_\eta^S e^{i\omega t}, \quad u_r^F = R U_\eta^F e^{i\omega t}, \quad p = \mu^S P e^{i\omega t} \quad (5)$$

于是, 引入无量纲量和参数:

$$\left\{ \begin{aligned} \eta = \frac{r}{R}, \quad \lambda = \frac{R\omega}{V^S}, \quad V^S = \sqrt{\frac{\mu^S}{\rho^S}}, \quad \bar{S}_v = \frac{R S_v}{V^S \rho^S}, \\ \rho^{FS} = \frac{\rho^F}{\rho^S}, \quad \delta = \frac{d}{R}, \quad \eta_0 = 1 + \delta, \\ T_\sigma = \frac{\tau_\sigma^\alpha V^S}{R}, \quad T_\varepsilon = \frac{\tau_\varepsilon^\alpha V^S}{R}. \end{aligned} \right. \quad (6)$$

则控制方程式(4)可化为:

$$\left\{ \begin{aligned} (1 + (i\lambda)^\alpha T_\sigma) \frac{d}{d\eta} \left[\frac{1}{\eta} \frac{d}{d\eta} (\eta U_\eta^S) \right] - \frac{1 - 2\nu^S}{2(1 - \nu^S)} \times \\ (1 + (i\lambda)^\alpha T_\varepsilon) \left(n^S \frac{dP}{d\eta} - \lambda^2 U_\eta^S - \right. \\ \left. i\lambda \bar{S}_v (U_\eta^F - U_\eta^S) \right) = 0 \\ \frac{dP}{d\eta} - \frac{\lambda^2 \rho^{FS}}{n^F} U_\eta^F + \frac{i\lambda \bar{S}_v}{n^F} (U_\eta^F - U_\eta^S) = 0 \\ \frac{1}{\eta} \frac{d}{d\eta} [\eta (n^S U_\eta^S + n^F U_\eta^F)] = 0 \end{aligned} \right. \quad (7)$$

由式(7)第三式, 得:

$$n^S U_\eta^S + n^F U_\eta^F = \frac{C_1}{\eta} \quad (8)$$

其中, C_1 为待定系数。

由式(7)前两式, 可得:

$$\frac{d}{d\eta} \left[\frac{1}{\eta} \frac{d}{d\eta} (\eta U_\eta^S) \right] - D_1 U_\eta^S + D_2 U_\eta^F = 0 \quad (9)$$

其中:

$$\left\{ \begin{aligned} D_1 &= \frac{(1 - 2\nu^S)(\bar{S}_v \lambda i - n^F \lambda^2)}{\chi(2 - 2\nu^S)n^F} \\ D_2 &= \frac{(1 - 2\nu^S)(\bar{S}_v \lambda i - n^S \rho^{FS} \lambda^2)}{\chi(2 - 2\nu^S)n^F} \end{aligned} \right. \quad (10)$$

令 $h^2 = D_1 + n^S D_2 / n^F$, 利用式(8)、式(9)可进一步化为:

$$\frac{d^2 U_\eta^S}{d\eta^2} + \frac{1}{\eta} \frac{d U_\eta^S}{d\eta} - \frac{U_\eta^S}{\eta^2} - h^2 U_\eta^S + \frac{D_2 C_1}{n^F} \frac{1}{\eta} = 0 \quad (11)$$

利用边界条件 $U_\eta^S \rightarrow 0 (\eta \rightarrow \infty)$, 此方程有解:

$$U_\eta^S = C_2 K_1(h\eta) + C_1 \frac{D_2}{n^F h^2} \frac{1}{\eta} \quad (12)$$

其中: $K_n(\cdot)$ 为第二类变形贝塞尔函数; C_2 为待定系数。从而, 由式(8)可得:

$$U_\eta^F = \frac{C_1}{\eta n^F} \left(1 - \frac{n^S D_2}{n^F h^2} \right) - \frac{n^S}{n^F} C_2 K_1(h\eta) \quad (13)$$

由式(7)第二式得:

$$P = D_3 C_1 \ln \eta - \frac{D_4 C_2}{h} K_0(h\eta) + C_3 \quad (14)$$

其中, C_3 为待定系数, 且:

$$\left\{ \begin{aligned} D_3 &= \frac{D_2 \bar{S}_v \lambda i}{h^2 (n^F)^2} + \frac{\lambda^2 \rho^{FS} - \lambda \bar{S}_v i}{(n^F)^2} \left(1 - \frac{n^S D_2}{n^F h^2} \right) \\ D_4 &= \frac{\bar{S}_v \lambda i}{n^F} - \frac{n^S (\lambda^2 \rho^{FS} - \lambda \bar{S}_v i)}{(n^F)^2} \end{aligned} \right. \quad (15)$$

为满足边界条件 $\eta \rightarrow \infty$ 时, 土体孔隙水压力 $P \rightarrow 0$, 引入大数 K , 使得 $P = 0 (K = 0)$, 由此得:

$$C_3 = -D_3 C_1 \ln K + \frac{D_4 C_2}{h} K_0(hK) \approx -D_3 C_1 \ln K \quad (16)$$

从而,

$$P = D_3 C_1 \ln \frac{\eta}{K} - \frac{D_4 C_2}{h} K_0(h\eta) \quad (17)$$

由本构关系式(2)得土骨架有效应力:

$$T_\pi^{SE} = \frac{2\mu^S \chi}{1 - 2\nu^S} e^{i\omega t} (A_1 C_1 - A_2 C_2) \quad (18)$$

其中:

$$\begin{aligned} A_1 &= -(1-2\nu^S)D_2/n^F h^2 \eta^2, \\ A_2 &= (1-\nu^S)hK_0(h\eta) + (1-2\nu^S)K_1(h\eta)/\eta, \\ \chi &= [1+T_\sigma(i\lambda)^\alpha]/[1+T_\varepsilon(i\lambda)^\alpha]. \end{aligned}$$

而饱和粘弹性土径向总应力为:

$$\begin{aligned} T_{rr}^T &= e^{i\omega t} \mu^S \bar{T}_{rr}^T = e^{i\omega t} \mu^S \cdot \\ &\left\{ -A_3 C_1 + \left[A_4 K_0(h\eta) - \frac{2K_1(h\eta)\chi}{\eta} \right] C_2 \right\} \quad (19) \end{aligned}$$

其中:

$$\begin{aligned} A_3 &= D_3 \ln(\eta/K) + 2\chi D_2/n^F h^2 \eta^2, \\ A_4 &= D_4/h - 2(1-\nu^S)h\chi/(1-2\nu^S). \end{aligned}$$

2 土-衬砌系统的简谐振动

记衬砌弹性模量和泊松比分别为 E^L 和 ν^L , 质量密度为 ρ^L , 将衬砌视为轴对称平面问题, 设其径向位移为 $u_r^L = R U_\eta^L e^{i\omega t}$, 则控制方程为:

$$\frac{d^2 U_\eta^L}{d\eta^2} + \frac{1}{\eta} \frac{dU_\eta^L}{d\eta} - \frac{U_\eta^L}{\eta^2} - q^2 U_\eta^L = 0 \quad (20)$$

其中:

$$\begin{cases} q^2 = -\rho^{LS} G^{SL} \frac{1-2\nu^L}{2(1-\nu^L)} \lambda^2 \\ \rho^{LS} = \frac{\rho^L}{\rho^S}, G^{SL} = \frac{\mu^S}{G^L} = \frac{2(1+\nu^L)\mu^S}{E^L} \end{cases} \quad (21)$$

可容易得到式(20)的解:

$$U_\eta^L = C_5 I_1(q\eta) + C_6 K_1(q\eta) \quad (22)$$

其中, C_5 和 C_6 为待定系数,

衬砌的径向应力为:

$$T_{rr}^L = e^{i\omega t} \frac{2G^L}{1-2\nu^L} \bar{T}_{rr}^L = e^{i\omega t} \frac{2G^L}{1-2\nu^L} (A_5 C_5 - A_6 C_6) C_5 \quad (23)$$

其中:

$$\begin{aligned} A_5 &= (1-\nu^L)qI_0(q\eta) - (1-2\nu^L)I_1(q\eta)/\eta, \\ A_6 &= (1-\nu^L)qK_0(q\eta) + (1-2\nu^L)K_1(q\eta)/\eta. \end{aligned}$$

饱和粘弹性土-衬砌系统的耦合振动应满足如下无量纲形式的边界条件和连接条件:

$$\begin{cases} \bar{T}_{rr}^L = (1-2\nu^L)(1+\nu^L) \frac{q_0}{E^L}, & \eta = 1 \\ \frac{2G^L}{(1-2\nu^L)\mu^S} \bar{T}_{rr}^L = \bar{T}_{rr}^T, U_\eta^L = U_\eta^S, & \eta = \eta_0 \end{cases} \quad (24)$$

若土-衬砌接触面处孔隙水自由渗透, 则:

$$P = 0, \eta = \eta_0 \quad (25)$$

若土-衬砌接触面处孔隙水不渗透, 则:

$$U_\eta^F = U_\eta^S, \eta = \eta_0 \quad (26)$$

利用边界和连接条件式(24)、条件式(25)或条件式(24)、条件式(26)可得到待定系数 C_1 、 C_2 、 C_5 和 C_6 满足的线性代数方程组, 由此可确定这些系数, 其具体表达式见附录。

至此, 得到透水或不透水条件下饱和粘弹性土-弹性衬砌系统简谐振动的解析解。

3 数值结果和讨论

参考文献[20-21], 取如下参数:

$$\begin{aligned} n^F &= 0.4, n^S = 0.6, \nu^S = 0.35, \bar{S}_v = 0.05, \nu^L = 0.25, \\ \delta &= 0.001, \rho^{FS} = 0.5, \frac{2(1+\nu^L)q_0}{E^L} = 1.0, \rho^{LS} = 1.5, \\ G^{SL} &= 0.01, \alpha = 0.5, T_\sigma/T_\varepsilon = 3, T_\varepsilon = 10. \end{aligned} \quad (27)$$

首先考察参数 K 的影响, 图2给出了不同参数 K 时, 在 $\eta = 1.5$ 处饱和粘弹性土体无量纲径向位移幅值 $|U| = \{[\text{Re}(U_\eta^S)]^2 + [\text{Im}(U_\eta^S)]^2\}^{1/2}$ 和无量纲孔隙水压力幅值 $|P^*| = [\mu^S([\text{Re}(P)]^2 + [\text{Im}(P)]^2)^{1/2}]/\mu^L$ 随无量纲频率 λ 的变化规律。可见, 随着参数 K 的增加, 位移幅值 $|U|$ 和孔压幅值 $|P^*|$ 随频率 λ 最终趋于某一极限状态。计算发现, $K=60$ 与 $K=50$ 的位移幅值和孔压幅值几乎相同, 对系统响应无影响。因此, 在以后的计算分析中, 均取 $K=60$ 。

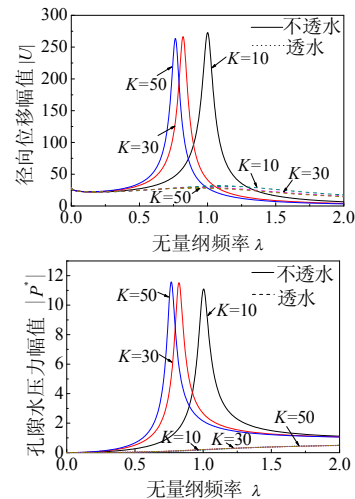


图2 参数 K 对径向位移幅值和孔隙水压力幅值的影响
Fig.2 Influence of the parameter K on the amplitudes of the radial displacement and pore water pressure

除参数 δ 外, 其余参数仍取式(27)中数据, 图3给出了在不同衬砌厚度 δ 下, $\alpha = 0.5$ 时分数导数型饱和粘弹性土-衬砌系统径向位移幅值 $|U|$ 随频率 λ 的变化, 而图4和图5分别给出了经典饱和弹性土 ($T_\sigma = 0, T_\varepsilon = 0$ 或 $\alpha = 0$) 和经典饱和粘弹土

($\alpha=1$)-衬砌系统径向位移幅值 $|U|$ 随频率 λ 的变化。可见,对于不透水系统,随着衬砌厚度 δ 的增加,三种情形下的基频均明显增大,而在透水情形下,厚度 δ 对系统响应影响较弱。同时,不同系统中,衬砌厚度 δ 对径向位移幅值 $|U|$ 的影响有本质的差异,随着衬砌厚度 δ 增加,饱和和弹性土-衬砌系统的位移幅值 $|U|$ 的峰值明显减小,分数导数型饱和和粘弹性土($\alpha=0.5$)-衬砌系统的位移幅值 $|U|$ 峰值

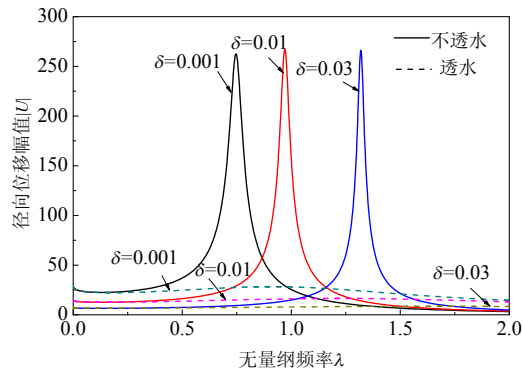


图3 分数导数粘弹性土中衬砌厚度 δ 对径向位移幅值的影响

Fig.3 Influence of lining thickness δ on the amplitudes of the radial displacement in viscoelastic soil with fraction derivative

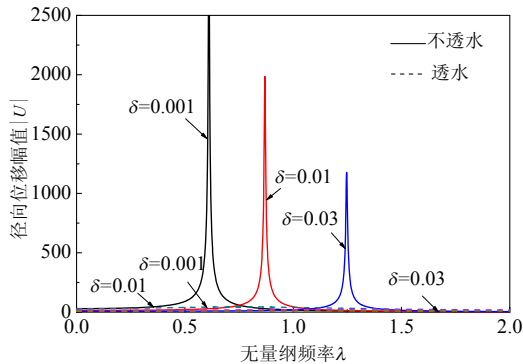


图4 弹性土体中衬砌厚度 δ 对径向位移幅值的影响

Fig.4 Influence of lining thickness δ on the amplitudes of the radial displacement in elastic soil

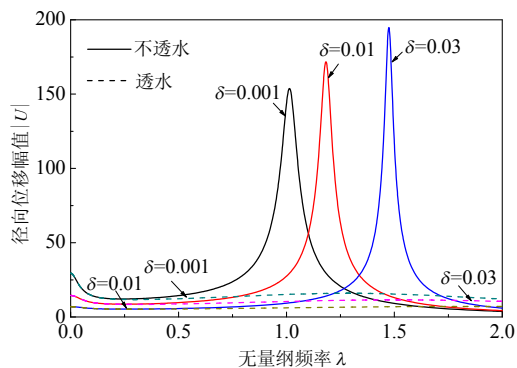


图5 经典粘弹性土中衬砌厚度 δ 对径向位移幅值影响

Fig.5 Influence of lining thickness δ on the amplitudes of the radial displacement in classic viscoelastic soil

变化很小,而经典饱和和粘弹土体-衬砌系统的位移幅值 $|U|$ 峰值逐渐增大。另外,数值计算表明土骨架和孔隙水相互作用系数 $\bar{S}_v$ 和土体泊松比 ν^s 对径向位移幅值 $|U|$ 几乎没有影响。

如图6和图7给出了 $T_e=10$, T_σ/T_e 分别为 $T_\sigma/T_e=3$ 和 $T_\sigma/T_e=0.5$ 时,分数导数阶数 α 对径向位移幅值 $|U|$ 的影响,而图8给出了当 $T_e=0$, $T_\sigma=30$ 时,分数导数阶数 α 对径向位移幅值 $|U|$ 的

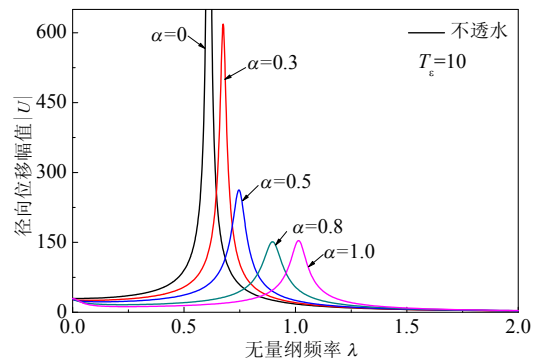


图6 $T_\sigma/T_e=3$ 时分数导数阶数 α 对径向位移幅值的影响

Fig.6 Influence of the order α of fractional derivative on the radial displacement amplitudes when $T_\sigma/T_e=3$

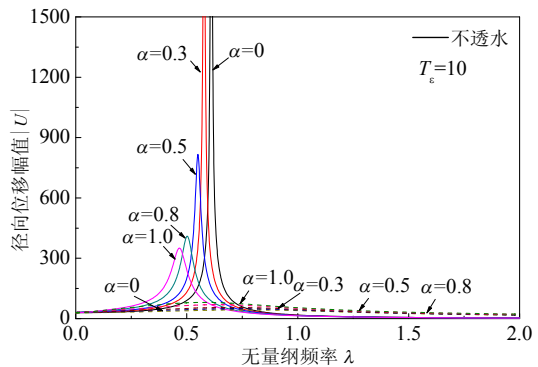


图7 $T_\sigma/T_e=0.5$ 时分数导数阶数 α 对径向位移幅值的影响

Fig.7 Influence of the order α of fractional derivative on the radial displacement amplitudes when $T_\sigma/T_e=0.5$

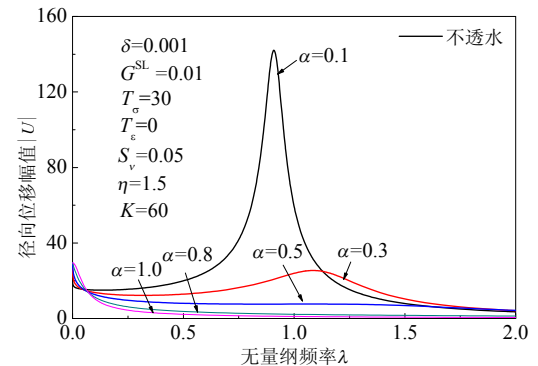


图8 $T_e=0$, $T_\sigma=30$ 时分数导数阶数 α 对径向位移幅值的影响

Fig.8 Influence of the order α of fractional derivative on the radial displacement amplitudes when $T_e=0$ and $T_\sigma=30$

影响。可见, 阶数 α 对位移幅值 $|U|$ 有显著影响, 随着阶数 α 的增加, 位移幅值 $|U|$ 的峰值明显减小, 饱和弹性土体($\alpha=0$)的峰值最大, 而饱和经典粘弹性土体($\alpha=1$)峰值达到最小值。并且, 随着 α 的增加, 当 $T_\sigma/T_\varepsilon > 1$ 时, 基频逐渐增加, 而当 $T_\sigma/T_\varepsilon < 1$ 时, 基频逐渐减小。当 $T_\varepsilon=0$, $T_\sigma=30$ 时, 阶数 α 对位移幅值 $|U|$ 影响较复杂。当 $\alpha > 0.3$ 时, 位移幅值 $|U|$ 峰值不明显, 共振效应减弱。

图 9~图 11 分别给出了上述三种情形下孔隙水压力幅值 $|P^*|$ 随频率 λ 的变化。可见, 分数导数阶数 α 对孔隙水压力幅值 $|P^*|$ 的影响类似于对位移幅值 $|U|$ 的影响。当 $T_\sigma/T_\varepsilon > 1$ 时, 随阶数 α 的增加, 系统基频增加, 孔压幅值 $|P^*|$ 峰值先逐渐减小, 而后逐渐增加。饱和弹性土体($\alpha=0$)的峰值最大, 但饱和经典粘弹性土体($\alpha=1$)的孔压幅值峰值大于分数导数模型阶数 $\alpha=0.8$ 时的孔压幅值峰值; 当 $T_\sigma/T_\varepsilon < 1$ 时, 随阶数 α 的增加, 孔压幅值和基频逐渐减小; 当 $T_\varepsilon=0$, $T_\sigma=30$, $\alpha > 0.3$ 时, 孔压幅值峰值不明显。

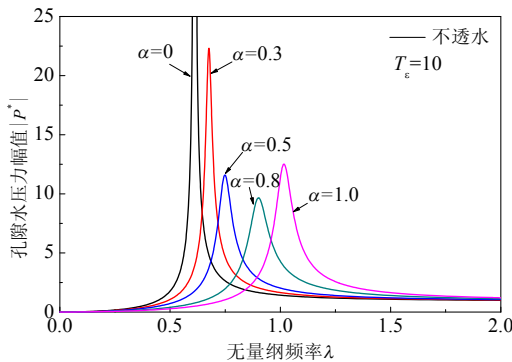


图 9 $T_\sigma/T_\varepsilon=3$ 时分数导数阶数 α 对孔隙水压力幅值的影响
Fig.9 Influence of the order α of fractional derivative on pore water pressure when $T_\sigma/T_\varepsilon=3$

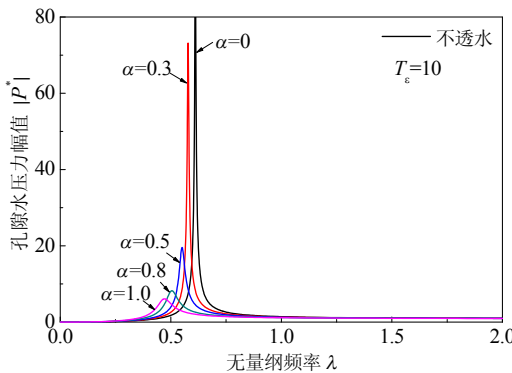


图 10 $T_\sigma/T_\varepsilon=0.5$ 时分数导数阶数 α 对孔隙水压力幅值的影响
Fig.10 Influence of the order α of fractional derivative on pore water pressure when $T_\sigma/T_\varepsilon=0.5$

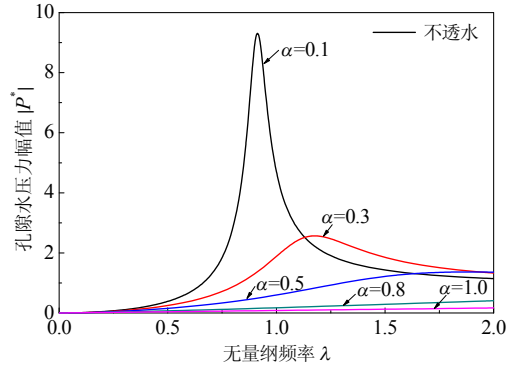


图 11 $T_\varepsilon=0$, $T_\sigma=30$ 时分数导数阶数 α 对孔隙水压力幅值的影响

Fig.11 Influence of the order α of fractional derivative on pore water pressure when $T_\varepsilon=0$ and $T_\sigma=30$

其它参数仍取式(27), 图 12 和图 13 分别给出了粘弹性参数比 T_σ/T_ε 对径向位移幅值 $|U|$ 和孔隙水压力幅值 $|P^*|$ 的影响。可见, 随着粘弹性参数比 T_σ/T_ε 的增加, 系统响应的基频增大, 响应幅值逐渐减小。

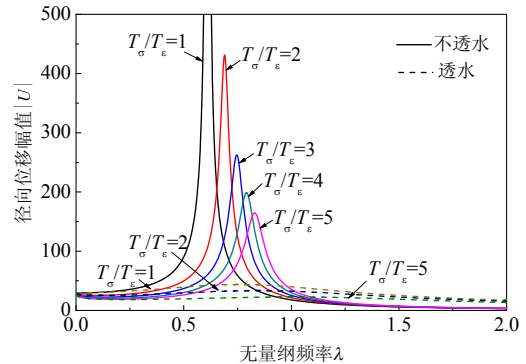


图 12 $T_\varepsilon=10$ 时, T_σ/T_ε 对径向位移幅值的影响
Fig.12 Influence of parameters T_σ/T_ε on the radial displacement amplitudes when $T_\varepsilon=10$

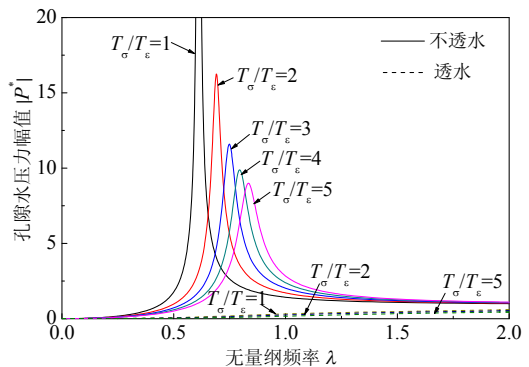


图 13 $T_\varepsilon=10$ 时, T_σ/T_ε 对孔隙水压力幅值的影响
Fig.13 Influence of parameters T_σ/T_ε on pore water pressure when $T_\varepsilon=10$

4 结论

本文利用分数导数描述土骨架粘弹性本构关

系,在频率域内研究了饱和粘弹性土-深埋圆形隧洞衬砌系统的简谐动力响应,讨论了几何和材料参数等对系统响应的影响,得到如下结论:

(1) 不透水和透水情形下深埋圆形隧洞饱和粘弹性土-衬砌系统动力学特性具有较大差异。而且,土骨架与孔隙水相互作用系数 $\bar{S}_v$ 和土体泊松比 ν^s 对系统响应的影响较弱。

(2) 导数阶数 α 和材料参数 τ_ϵ^α 、 τ_σ^α 对系统响应有显著影响,可通过阶数 α 和材料参数 τ_ϵ^α 、 τ_σ^α 等的选取更好地模拟饱和粘弹性土体的力学行为。分数导数型饱和粘弹性模型、经典饱和和弹性($T_\sigma=0$, $T_\epsilon=0$)/粘弹土模型($\alpha=1$)的系统响应差异很大。

(3) 随着衬砌厚度 δ 增加,不透水情形下系统响应的基频明显增大,而对透水情形下系统响应影响较小。同时,衬砌厚度 δ 对径向位移幅值 $|U|$ 的影响较为复杂。

(4) 当 $T_\sigma/T_\epsilon > 1$ 时,随阶数 α 的增加,系统基频增加,位移幅值 $|U|$ 和孔压幅值 $|P^*|$ 的峰值先逐渐减小,而后逐渐增加;当 $T_\sigma/T_\epsilon < 1$ 时,随阶数 α 的增加,位移幅值 $|U|$ 和孔压幅值 $|P^*|$ 的峰值和基频逐渐减小。

(5) 随着粘弹性参数比 T_σ/T_ϵ 的增加,系统响应的基频增大,响应幅值逐渐减小。

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附录:

待定系数 C_1 、 C_2 、 C_5 和 C_6 的具体表达式为:

$$\begin{cases} C_2 = \frac{\frac{b_2}{a_2} \left(d_1 - \frac{a_1}{a_2} d_2 \right) - \frac{d_2}{a_2} \left(b_1 - \frac{a_1}{a_2} b_2 \right)}{\left(b_1 - \frac{a_1}{a_2} b_2 \right) \left(d_4 - \frac{e_2}{e_1} d_3 \right) - \left(d_1 - \frac{a_1}{a_2} d_2 \right) \left(b_4 - \frac{e_2}{e_1} b_3 \right)} Q, & C_1 = -\frac{e_1}{e_2} C_2 \\ C_5 = \frac{\frac{d_2}{a_2} \left(b_4 - \frac{e_2}{e_1} b_3 \right) - \frac{b_2}{a_2} \left(d_4 - \frac{e_2}{e_1} d_3 \right)}{\left(b_1 - \frac{a_1}{a_2} b_2 \right) \left(d_4 - \frac{e_2}{e_1} d_3 \right) - \left(d_1 - \frac{a_1}{a_2} d_2 \right) \left(b_4 - \frac{e_2}{e_1} b_3 \right)} Q, & C_6 = \frac{Q}{a_2} - \frac{a_1}{a_2} C_5 \end{cases} \quad (A1)$$

其中:

$$\begin{cases} a_1 = \frac{2}{1-2\nu^L} [(1-\nu^L)qI_0(q) - (1-2\nu^L)I_1(q)] \\ a_2 = -\frac{2}{1-2\nu^L} [(1-\nu^L)qK_0(q) + (1-2\nu^L)K_1(q)], & d_1 = I_1(q\eta_0) \\ b_1 = \frac{2}{1-2\nu^L} \left[(1-\nu^L)qI_0(q\eta_0) - \frac{(1-2\nu^L)}{\eta_0} I_1(q\eta_0) \right], & d_2 = K_1(q\eta_0) \\ b_2 = -\frac{2}{1-2\nu^L} \left[(1-\nu^L)qK_0(q\eta_0) + \frac{(1-2\nu^L)}{\eta_0} K_1(q\eta_0) \right], & d_3 = -\frac{D_2}{n^F h^2 \eta_0} \\ b_3 = G^{SL} \left[D_3 \ln(\eta_0 / K) + \frac{2\chi D_2}{n^F h^2 \eta_0^2} \right], & d_4 = -K_1(h\eta_0) \\ b_4 = -G^{SL} \left\{ \left[\frac{D_4}{h} - \frac{2(1-\nu^S)\chi h}{1-2\nu^S} \right] K_0(h\eta_0) - \frac{2\chi K_1(h\eta_0)}{\eta_0} \right\}, & Q = \frac{q_0}{G^L} \end{cases} \quad (A2)$$

其中, 不渗透时:

$$e_1 = \frac{1}{n^F \eta_0} \left(1 - \frac{n^S D_2}{n^F h^2} \right) - \frac{D_2}{n^F h^2 \eta_0}, \quad e_2 = -\frac{K_1(h\eta_0)}{n^F} \quad (A3)$$

自由渗透时:

$$e_1 = D_3 \ln(\eta_0 / K), \quad e_2 = -\frac{D_4}{h} K_0(h\eta_0) \quad (A4)$$